

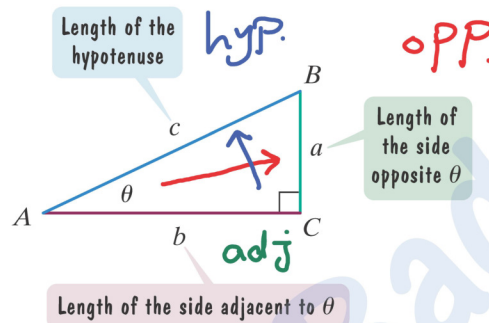
Right Triangle Trigonometry

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$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{a}{c}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{b}{c}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{a}{b}$$



Note

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

Example 1

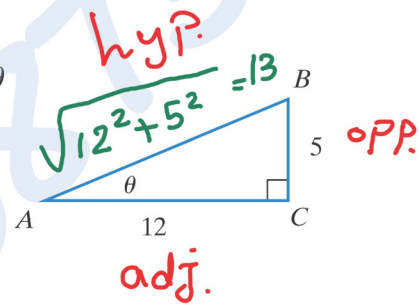
Find the value of each of the six trigonometric functions of θ

Solution

$$\sin \theta = \frac{5}{13}$$

$$\cos \theta = \frac{12}{13}$$

$$\tan \theta = \frac{5}{12}$$



Example 2

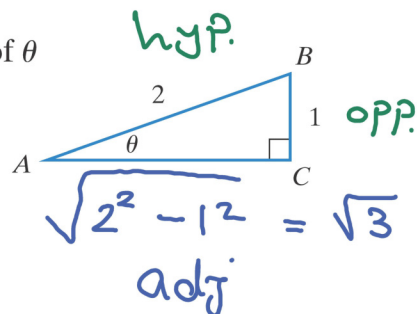
Find the value of each of the six trigonometric functions of θ

Solution

$$\sin \theta = \frac{1}{2}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

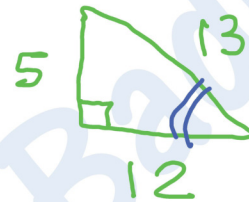
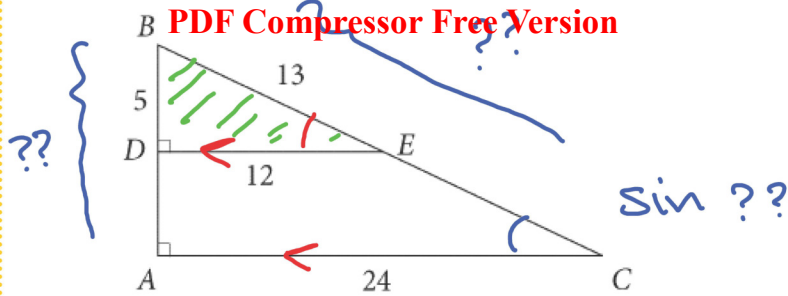
$$\tan \theta = \frac{1}{\sqrt{3}}$$



Example 3 SAT September 2020 International



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In triangle ABC , point E lies on line segment BC and point D lies on line segment AB . What is the sine of angle C ?

A) $\frac{5}{13}$

B) $2\left(\frac{5}{13}\right)$

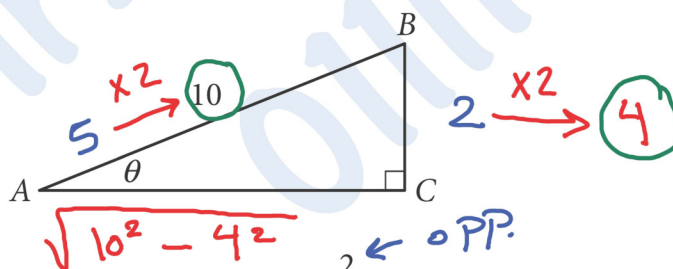
C) $\frac{12}{13}$

D) $2\left(\frac{12}{13}\right)$

Solution

$$\sin \angle BED = \sin C = \frac{5}{13}$$

Example 4 SAT April 2019 US



In the right triangle above, $\sin \theta = \frac{2}{5}$. If $AC = \sqrt{n}$, what is the value of n ?

Solution

$$\sqrt{84} \rightarrow n = 84$$

Example 5 EST March 2021

In the right triangle below, what is the length of BC?

A. $3 \sin 40$

B. $\frac{\sin 40}{3}$

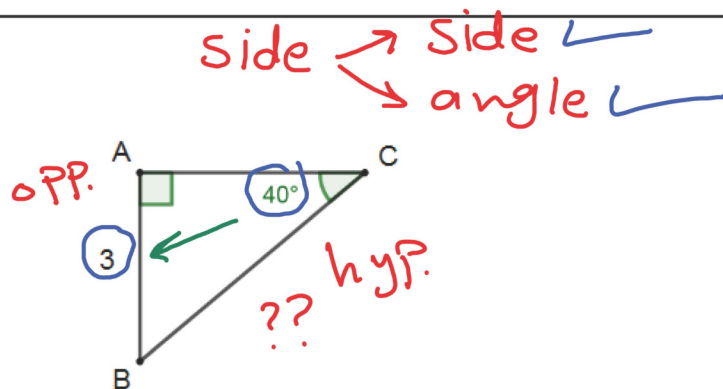
✓ C. $\frac{3}{\sin 40}$

D. $3 \cos 40$

Solution

$$\frac{\sin 40^\circ}{1} \neq \frac{3}{BC}$$

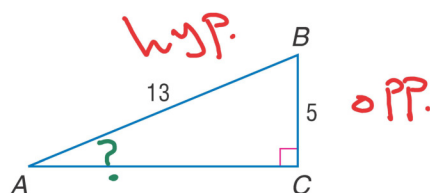
$$BC = \frac{3-1}{\sin 40^\circ} = \frac{3}{\sin 40^\circ}$$



Example 6

Find $m\angle A$

Solution

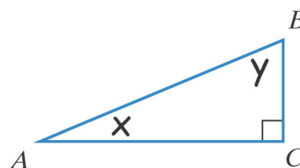


angle $\rightarrow$ Side
 $\rightarrow$ Side

$$\sin A = \frac{5}{13} \quad \angle A = \sin^{-1} \frac{5}{13} = 22.6$$

Rules

If $x + y = 90^\circ$ then $\sin x = \cos y$

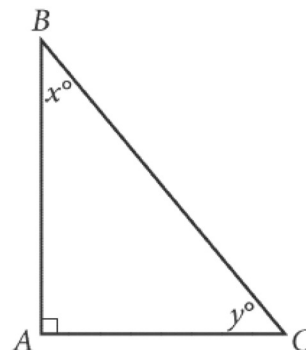


Example 7 SAT October 2020 US

In right triangle ABC above, $\sin(x^\circ) = \frac{5}{8}$. What is the

value of $\cos(y^\circ)$? $\frac{5}{8}$

Solution



Example 8 EST December 2020



If x and y are positive measures of acute angles, and $\sin(x - 20^\circ) = \cos(y + 12^\circ)$ what is a possible value of $x + y$?

$$\underline{x - 20} + \underline{y + 12} = 90$$

Solution

$$\sin x = \cos y$$

$$x + y = 90^\circ$$

$$(x + y) - 8 = 90$$

$$x + y = 90 + 8 = 98$$

Notes

- $\sin \theta = \cos(90^\circ - \theta)$ $\cos \theta = \sin(90^\circ - \theta)$

If θ is in radians, replace 90° with $\frac{\pi}{2}$

Example $\sin 72^\circ = \cos(90^\circ - 72^\circ) = \cos 18^\circ$

$$\cos \frac{\pi}{3} = \sin \left(\frac{\pi}{2} - \frac{\pi}{3} \right) = \sin \frac{\pi}{6}$$

$$\sin 52^\circ = \cos ??$$

$$\cos \frac{\pi}{3} = \sin ??$$

$$\begin{aligned} & \frac{\pi}{2} - \frac{\pi}{3} \\ &= \frac{\pi}{6} \end{aligned}$$

Example 9 EST October 2020



If $\sin A = \cos \frac{7\pi}{3}$, what is one possible value of A in radians?

- A. $\frac{\pi}{2}$
- B. $\frac{\pi}{3}$
- C. $\frac{\pi}{4}$
- ☒ D. $\frac{\pi}{6}$

Solution

$$360^\circ \rightarrow 2\pi$$

$$\begin{aligned} A &= \frac{\pi}{2} - \frac{7\pi}{3} = -\frac{11}{6}\pi + 2\pi \\ &= \frac{\pi}{6} \end{aligned}$$

Example 10 SAT December 2017 US



Which of the following is equivalent to $\tan(x^\circ)$?

☒ A) $\frac{\sin(90^\circ - x^\circ)}{\cos(90^\circ - x^\circ)} = \frac{\cos x}{\sin x}$

Solution

$$\frac{\sin x}{\cos x}$$

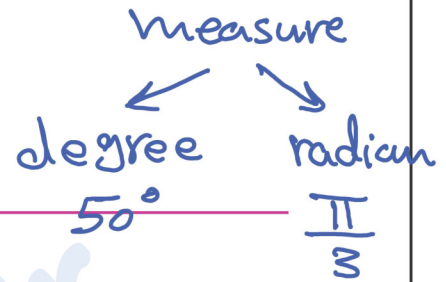
☒ B) $\frac{\cos(90^\circ - x^\circ)}{\sin(90^\circ - x^\circ)} = \frac{\sin x}{\cos x} = \tan x$

C) $-\tan(90^\circ - x^\circ)$

☒ D) $\sin(x^\circ) - \cos(x^\circ)$

Conversion between Degrees and Radians

1. To convert degrees to radians, multiply degrees by $\frac{\pi}{180^\circ}$
2. To convert radians to degrees, put $\pi = 180^\circ$



Example 11

Convert each angle in degrees to radians:

- a. 30°
- b. 90°

Solution

$$a) 30^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{6}$$

$$b) 90^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{2}$$

Example 12

Convert each angle in radians to degrees:

$$a. \frac{\pi}{3}$$

$$b. \frac{7\pi}{3}$$

Solution

$$\frac{\pi}{3} = \frac{180^\circ}{3} = 60^\circ$$

$$\frac{7\pi}{3} = \frac{7 \cdot 180^\circ}{3} = 420^\circ = 420^\circ - 360^\circ = 60^\circ$$

Example 13 SAT March 2018 US

An angle measure of 540 degrees was written in radians as $x\pi$. What is the value of x ?

Solution

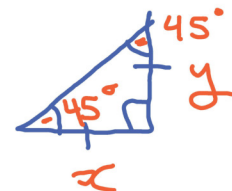
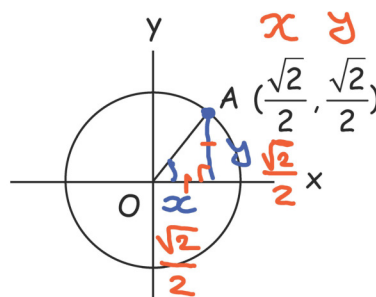
$$\frac{540}{180} \cdot \frac{\pi}{180} = 3\pi \rightarrow x = 3$$

91

Example 14 EST August 2021



In the xy -plane, a circle has its center at the origin O and a radius of $\frac{4\pi}{a}$ units. The measure of $\angle AOx$ is $\frac{4\pi}{a}$ radians. What is the value of a ?



Solution

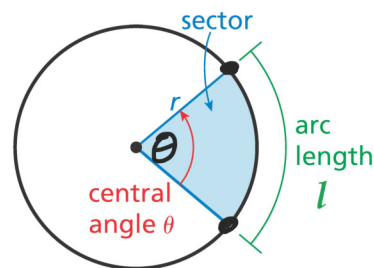
$$45^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{4}$$

$$\frac{\pi}{4} = \frac{4\pi}{a} \rightarrow \frac{1}{4} = \frac{4}{a} \rightarrow a = 4 \cdot 4 = 16$$

Arc Length and Area of a Sector

Arc length: $l = \frac{\theta^\circ}{360^\circ} \cdot 2\pi r$ ← Circumference

Area of sector: $A = \frac{\theta^\circ}{360^\circ} \cdot \pi r^2$ ← area



Example 15 EST October 2020



What is the area of the sector defined by a central angle of 54° in the circle of equation $x^2 + y^2 - 8x + 12y - 12 = 0$?

Solution

$$\theta = 54^\circ$$

- A. 15π
- ✓ B. 9.6π
- C. 5.4π
- D. 2.4π

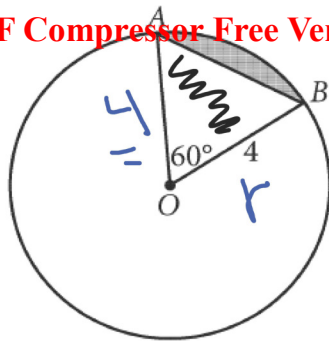
$$x^2 + y^2 - 8x + 12y = 12$$

$$r = \sqrt{4^2 + (-6)^2 + 12} = 8$$

$$\text{Area} = \frac{\theta}{360} \cdot \pi r^2 = \frac{54}{360} \cdot \pi (8)^2 = 9.6\pi$$

Example 16 SAT October 2020 US

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area shaded region = area of sector
-
area of Δ

In the figure above, the circle has center O. What is the area of the shaded region?

A) $\frac{8\pi}{3}$

B) $\frac{1}{3}(12\sqrt{3} - 4\pi)$

C) $\frac{1}{3}(8\pi - 8\sqrt{3})$

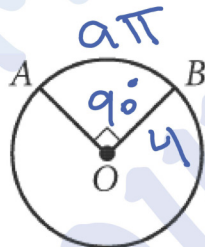
✓ D) $\frac{1}{3}(8\pi - 12\sqrt{3}) = 1.44$

Solution

$$= \frac{60}{360} \cdot \pi (4)^2 - \frac{1}{2} \cdot 4 \cdot 4 \cdot \sin 60^\circ$$

$$= 1.44$$

Example 17 SAT December 2016 US



In the circle above, O is the center and $OB = 4$. If the length of arc $\widehat{AB}$ is $a\pi$, where a is a constant, what is the value of a ?

Solution

$$\text{length of arc} = \frac{90}{360} \cdot 2\pi(4) = 2\pi$$

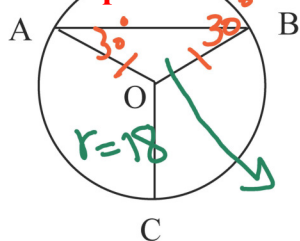
$$a = 2$$

Example 18

SAT May 2019 International



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$$180^\circ - (30^\circ + 30^\circ) = 120^\circ$$

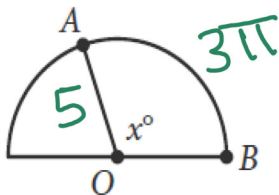
Point O is the center of the circle above, and the measure of $\angle OAB$ is 30° . If the length of $\overline{OC}$ is 18, What is the length of arc $\widehat{AB}$?

Solution

$$\begin{aligned} \text{length of } \widehat{AB} &= \frac{120^\circ}{360^\circ} \cdot 2\pi(18) \\ &= 12\pi \end{aligned}$$

Example 19

SAT April 2018



Segments $\overline{OA}$ and $\overline{OB}$ are radii of the semicircle above. Arc $\widehat{AB}$ has length 3π and $OA = 5$. What is the value of x ?

Solution

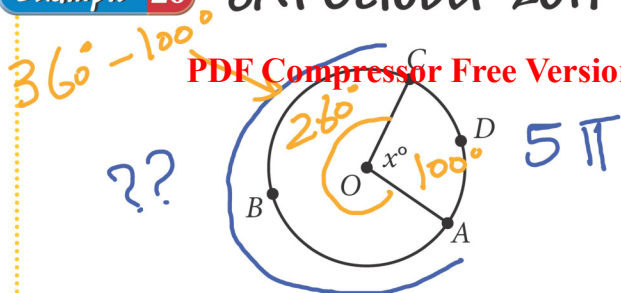
$$\begin{aligned} 3\pi &= \frac{x}{360^\circ} \cdot 2\pi(5) \\ 3 &= \frac{x}{36} \quad x = 3 \cdot 36 \\ &= 108 \end{aligned}$$

Example 20

SAT October 2017 US



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Solution

The circle above has center O , the length of arc $\widehat{ADC}$ is 5π , and $x = 100$. What is the length of arc $\widehat{ABC}$?

- A) 9π
- ☒ B) 13π
- C) 18π
- D) $\frac{13}{2}\pi$

$$\frac{\text{length of } \widehat{ABC}}{\text{length of } \widehat{ADC}} = \frac{260}{100}$$

Handwritten notes: 13, 5

$$\frac{\text{length of } \widehat{ABC}}{5\pi} = \frac{13}{5}$$

Handwritten notes: 13, 5

$$\text{length of } \widehat{ABC} = 13\pi$$