

Slope

graph

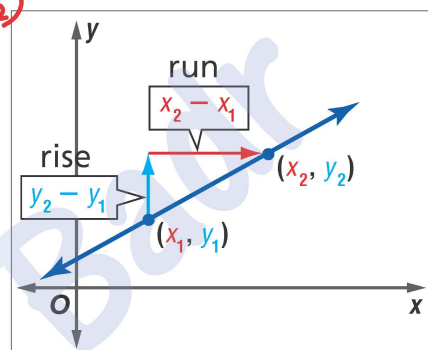


2 Points $(x_1, y_1), (x_2, y_2)$

$$\text{slope } (m) = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \leftarrow \begin{array}{l} \text{change in } y \\ \text{change in } x \end{array}$$

rise $\updownarrow$

run $\longleftrightarrow$



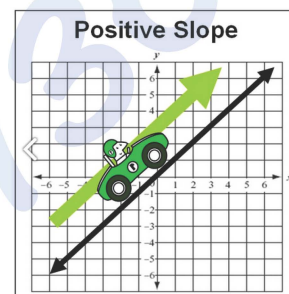
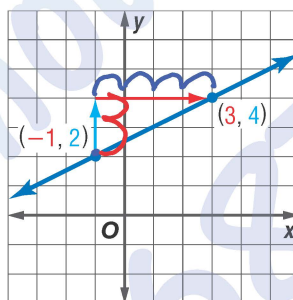
Example 1 Positive Slope

Find the slope of the line that passes through $(-1, 2)$ and $(3, 4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{3 - (-1)} = \frac{2}{4} = \frac{1}{2}$$

rise = 2 $\uparrow$
run = 4 $\rightarrow$

$$m = \frac{\text{rise}}{\text{run}} = \frac{2}{4} = \frac{1}{2}$$



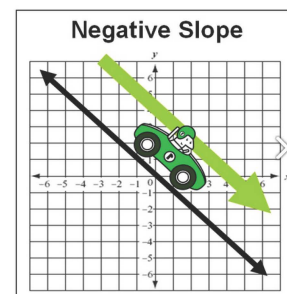
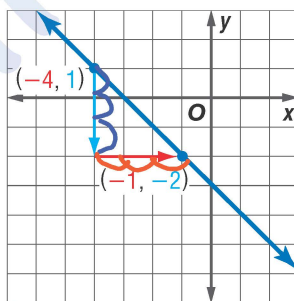
Example 2 Negative Slope

Find the slope of the line that passes through $(-1, -2)$ and $(-4, 1)$.

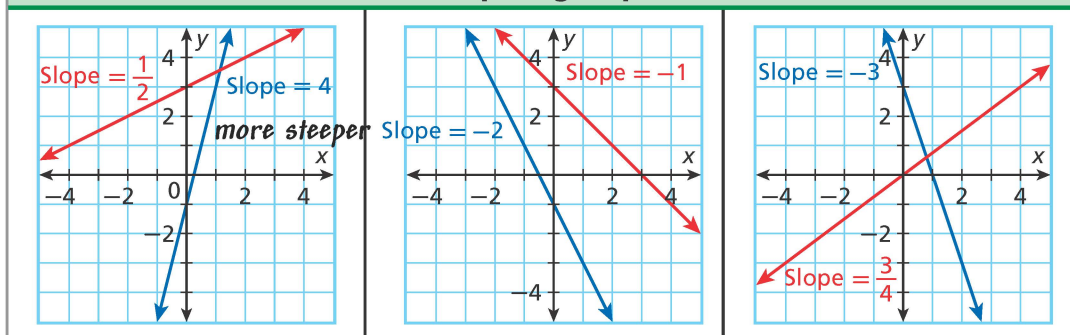
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-2)}{-4 - (-1)} = \frac{3}{-3} = -1$$

rise = -3 $\downarrow$
run = 3 $\rightarrow$

$$m = \frac{\text{rise}}{\text{run}} = \frac{-3}{3} = -1$$



Comparing Slopes

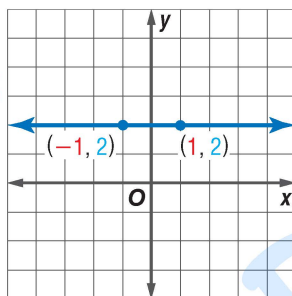


Example 3 Zero Slope

$$x_1 y_1 \quad x_2 y_2$$

Find the slope of the line that passes through (1, 2) and (-1, 2).

$$m = \frac{2-2}{-1-1} = 0 \quad // \text{X-axis}$$

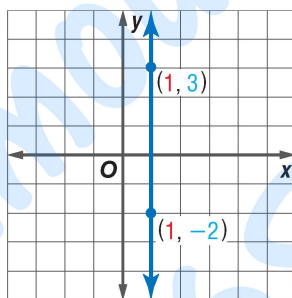


Horizontal line

Example 4 Undefined Slope

Find the slope of the line that passes through (1, -2) and (1, 3).

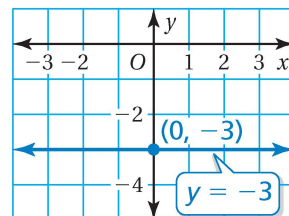
$$m = \frac{3-(-2)}{1-1} \text{ undefined}$$



Vertical line

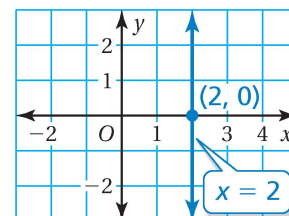
Notes

$y = -3$ line $//$ x-axis ($m = 0$)



Horizontal line

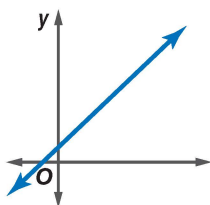
$x = 2$ line $//$ y-axis (m is undefined)



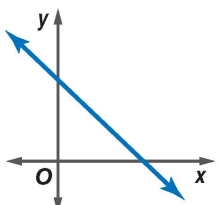
Vertical line

Summary

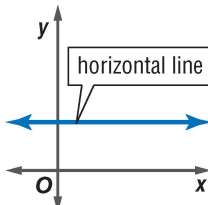
Positive Slope



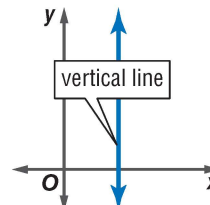
Negative Slope



Slope of 0



Undefined Slope



Note

If a line passes through the two points (x_1, y_1) , (x_2, y_2) and the origin point $(0,0)$

then $\frac{x_1}{x_2} = \frac{y_1}{y_2}$

Example 5 EST March 2021

In the xy-plane, the line determined by the points $(3, m)$ and $(m, 12)$ passes through the origin. Which of the following could be the value of m ?

- ✓ A. -6
B. 9
C. 1
D. 0

$$\frac{3}{m} \neq \frac{m}{12}$$

$$m^2 = 36$$

$$m = \pm \sqrt{36} = \pm 6$$

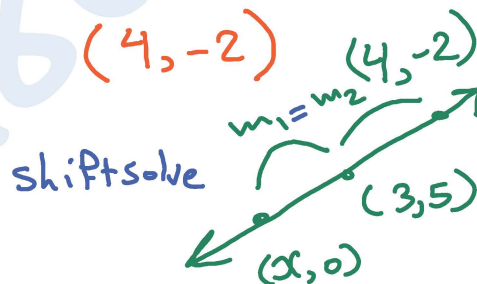
Solution**Example 6** EST May 2021

What is the x-intercept of the line passing through points $(3, 5)$ and $(4, -2)$?

- A. -7
B. $\frac{25}{7}$
✓ C. $\frac{26}{7}$
D. 26

$$\frac{5-0}{3-x} = \frac{-2-5}{4-3}$$

$$x = \frac{26}{7}$$

Solution**Example 7** EST June 2021

A line, having a slope of $-\frac{1}{5}$ passes through the points A $(2; \alpha-3)$ and B $(2\alpha+1; -4)$. What is the value of α ?

- A. -2
B. $-\frac{2}{3}$
C. $\frac{2}{3}$
D. 2

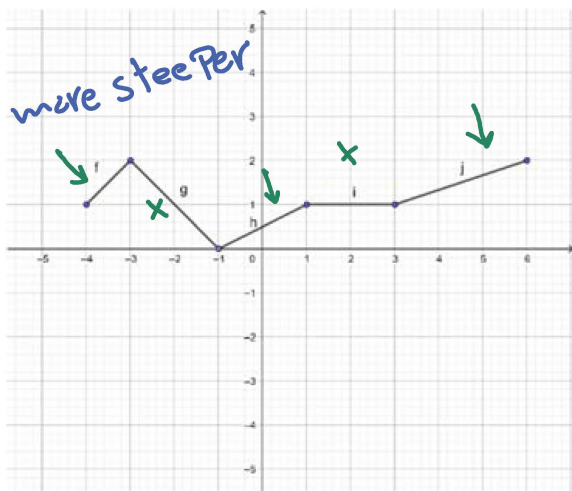
$$\frac{-4 - (\alpha - 3)}{2\alpha + 1 - 2} = -\frac{1}{5}$$

$$\alpha = -2$$

Solution

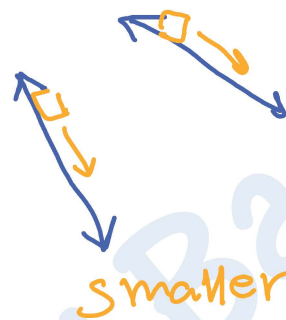
Example 8 EST March 2021

Which of the line segments in the figure below has the largest slope?



- A. f
- B. j
- C. h
- D. i

Solution



Find Slope using a Table

The points given in the table lie on a line. Find the slope of the line.

Choose two points from the table to find the changes in the x- and y-values.

$$\begin{aligned} \text{slope} &= \frac{\text{change in } y}{\text{change in } x} \\ &= \frac{9 - 12}{3 - 1} \\ &= \frac{-3}{2} \text{ or } -\frac{3}{2} \end{aligned}$$

The slope is $-\frac{3}{2}$.

inc.

x	1	3	5	7
y	12	9	6	3

dec.

(1, 12) > (3, 9)

x increase

x	-6	-2	2	6
y	-2	-1	0	1

Positive Slope

y increase

x increase

x	-4	0	4	8
y	-1	-2	-3	-4

Negative Slope

y decrease

Example 9 EST October 2020

X	2	5	-3
Y	-1	-7	?

If the values in the table above represent a linear relationship, what is the missing value?

Solution

$$m_1 = m_2$$

$$(2, -1) \quad (5, -7) \quad (-3, x)$$

$$\frac{-7 - (-1)}{5 - 2} = \frac{x - (-7)}{-3 - 5}$$

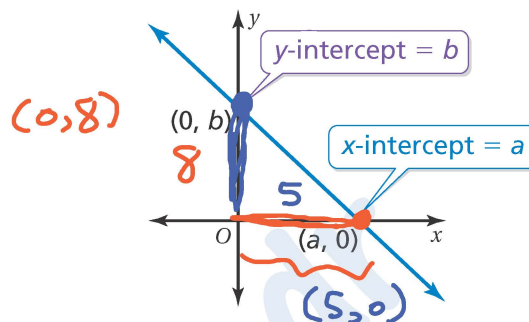
shift solve

$$x = 9$$

Intercepts

To find y -intercept put $x=0$ then find y

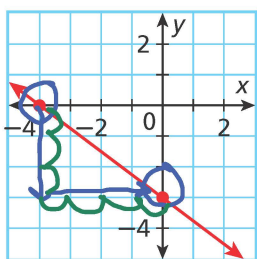
To find x -intercept put $y=0$ then find x



Example 10

Find x -intercept, y -intercept and the slope

A



Solution

$$x\text{int.} = -4$$

$$y\text{int.} = -3$$

$$(-4, 0), (0, -3)$$

$$m = \frac{-3 - 0}{0 - (-4)} = -\frac{3}{4}$$

rise $\downarrow -3$

run $\rightarrow 4$

$$\text{slope} = -\frac{3}{4}$$

B

$$3x - 2y = 12$$

Solution

$$m = \frac{-3}{-2} = \frac{3}{2}$$

$$x\text{int.} \rightarrow y = 0$$

$$3x = 12$$

$$x = \frac{12}{3} = 4$$

$$y\text{int.} \rightarrow x = 0$$

$$-2y = 12$$

$$y = \frac{12}{-2} = -6$$

Standard form

$$Ax + By = C$$

$$\text{Slope} = \frac{-\text{coeff. of } x}{\text{coeff. of } y} = -\frac{A}{B}$$

Example 11

SAT May 2021 US

The function f is a linear function. The y -intercept of the graph of $y = f(x)$ in the xy -plane is $(0, -12)$.

What is the y -intercept of the graph of

$$y = f(x) + 2?$$

A) $(0, -14)$

☒ B) $(0, -10)$

C) $(-2, -12)$

D) $(2, -12)$

2 units
to up

Solution

$$\begin{array}{r} \downarrow +2 \\ -10 \end{array}$$

$$\begin{array}{c} \uparrow -10 \\ -12 \end{array}$$

$$y = mx + b \quad \begin{matrix} \text{slope} \\ \text{y-int.} \end{matrix}$$

Example 12

EST October 2020 

The graph of the function h in the xy -plane contains the point $(2, 5)$ and has a y -intercept of -7 . The function g is defined by $g(x) = 3 - 2h(x)$. Which of the following points lie on the graph of g ?

- A. $(0, -7)$
- B. $(2, 17)$
- ✓ C. $(0, 17)$
- D. $(-7, 3)$

Solution

$$h \rightarrow -7$$

$$g \rightarrow ?$$

$$3 - 2(-7) = 17 \quad \text{y-int.}$$

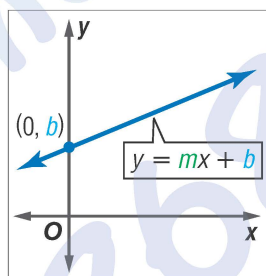
$$(0, 17)$$

Equation of the line

Slope-Intercept Form

$$y = mx + b$$

slope $\uparrow$ $\uparrow$ y -intercept

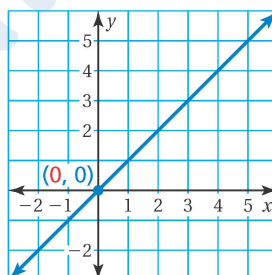


Note

If the line passes through the origin point then $b = 0$

$$y = mx$$

slope $\uparrow$



Example 13

Write an equation of the line whose slope is 3 and whose y -intercept is 5.

$$y = 3x + 5$$

Example 14

Find the slope and y-intercept of the graph of each linear equation.

a. $y = -4x - 2$

$m = -4$ $b = -2$

b. $y - 5 = \frac{3}{2}x$

$y = \frac{3}{2}x + 5$

$m = \frac{3}{2}$ $b = 5$

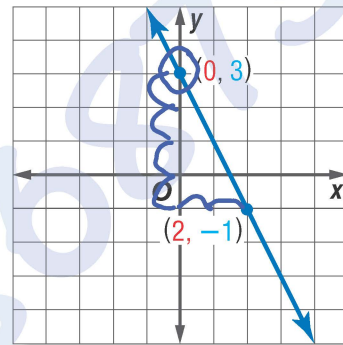
Example 15

Write an equation of the line shown in the graph.

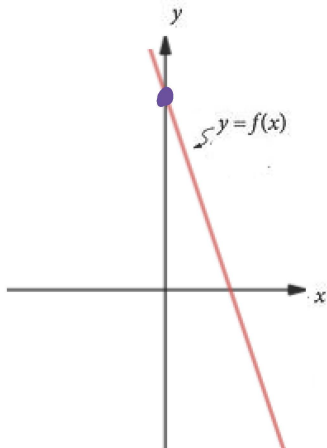
$m = \frac{-4}{2} = -2$

$b = 3$

$y = -2x + 3$

**Example 16**

EST December 2020



The graph shown above is that of a linear function f whose expression is given by $f(x) = cx + d$, where c and d are constants. Which of the following must be true about c and d ?

- A. $c = d$
- B. $c > d$
- ☒ C. $c < d$
- D. $c = 0$

Solution

$f(x) = cx + d$
 $mx + b$

slope
 $m \rightarrow -ve$
 $b \rightarrow +ve$
y-int.

$c \rightarrow -ve$
 $d \rightarrow +ve$

Example 17 Write an Equation Given Slope and One Point

Write an equation of a line that passes through (1, 5) with slope 2.

$$y = mx + b$$

$$2 \cdot 1 + b = 5$$

$$2 + b = 5$$

$$b = 5 - 2 = 3$$

$$y = 2x + 3$$

Write an Equation Given Two Points

Write the equation of the line passes through (-3, -1) and (6, -4)

By calculator Mode 5 1

Solution

$$m? \quad b?$$

$$m = \frac{-4 - (-1)}{6 - (-3)} = -\frac{1}{3}$$

$$y = -\frac{1}{3}x + b$$

$$-\frac{1}{3} \cdot 6 + b = -4$$

$$-2 + b = -4$$

$$b = -4 + 2 = -2$$

$$y = -\frac{1}{3}x - 2$$

-3	1	-1
6	1	-4

$$x = -\frac{1}{3} \rightarrow \text{slope}$$

$$y = -2 \rightarrow \text{y-intercept}$$

Example 18 EST December 2020

People who own a smartphone worldwide

Year	Number of People (in billions)
2016	2.3
2017	2.6
2018	2.9
2019	3.2
2020	3.5

The table above shows the number of people who owned a smartphone from the year 2016 to the year 2020. The number of people N (in billions) is a linear function of the number of years y from the year 2016. Which of the following expressions best describes N in terms of y ?

Solution

A. $N = 0.3y$

B. $N = 0.3y + 2016$

C. $N = 0.3y + 2.3$

D. $N = 3.3y + 2.3$

$$m = \frac{2.6 - 2.3}{1 - 0} = 0.3$$

start

Slope

2 points

$$(x_1, y_1), (x_2, y_2)$$

$$\text{Slope } (m) = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Ex: } (1, -2), (3, 5)$$

$$\text{Slope} = \frac{5 - (-2)}{3 - 1} = \frac{7}{2}$$

Angle (θ)

$$\text{Slope } (m) = \tan \theta$$

$$\text{Ex: } \theta = 45^\circ$$

$$\text{Slope} = \tan 45^\circ = 1$$

Slope intercept form

$$y = mx + b$$

↑ ↑
Slope y-intercept

$$\rightarrow \text{Ex: } y = 2x - 5$$

$$\text{Slope} = 2$$

$$\text{y-intercept} = -5$$

Standard form

$$Ax + By = C$$

$$\text{Slope} = \frac{-\text{coeff. of } x}{\text{coeff. of } y}$$

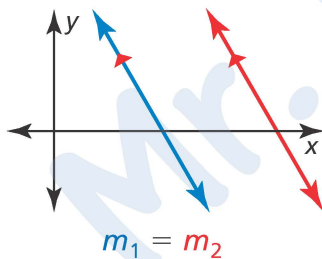
$$= \frac{-A}{B}$$

$$\text{Ex: } -3x + y - 8 = 0$$

$$\text{Slope } (m) = \frac{-(-3)}{1} = 3$$

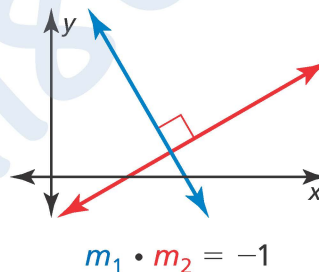
Parallel and perpendicular lines

Slopes of Parallel Lines



$$\text{If } m_1 = 5 \text{ then } m_2 = 5$$

Slopes of Perpendicular Lines



$$\text{If } m_1 = 2 \text{ then } m_2 = \frac{-1}{2} \text{ (sh2lb w 2lb)}$$

Example 19 EST June 2021

Line (A) passes through the points $(-\frac{5}{3}, 0)$ and $(0, -\frac{2}{3})$. Which of the following lines will never intersect with line (A)?

A. $y = -\frac{5}{2}x - 1$

$m = -\frac{5}{2}$

B. $y = \frac{5}{2}x - 1$

$m = \frac{5}{2}$

✓ C. $y = -\frac{2}{5}x - 1$

$m = -\frac{2}{5}$

D. $y = \frac{2}{5}x - 1$

Solution

$$m = \frac{-\frac{2}{3} - 0}{0 - (-\frac{5}{3})} = -\frac{2}{5}$$

Parallel

Example 20 EST June 2021

In the xy -plane, the equation of the line (d) passing through $A(-1,4)$ and perpendicular to line $3x-2y=5$ is:

A. $y = \frac{3}{2}x + \frac{11}{2}$

B. $y = -\frac{3}{2}x + \frac{5}{2}$

? C. $y = -\frac{2}{3}x + \frac{14}{3}$

? ✓ D. $y = -\frac{2}{3}x + \frac{10}{3}$

$$\downarrow \frac{-3}{-2} = \frac{3}{2}$$

$$m = -\frac{2}{3}$$

Solution

Plug in

$$-\frac{2}{3} \cdot -1 + \frac{10}{3} = 4$$

Example 21 EST December 2020

The straight line m has an equation $y = 3x$. The point A of coordinates $(1,3)$ is on m . The line p is perpendicular to m at point A . Which of the following points is on p ?

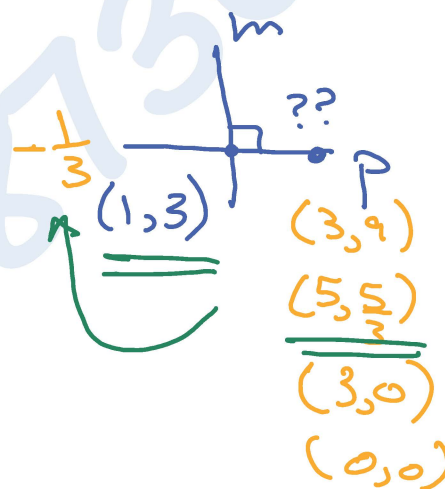
A. $(3, 9)$

✓ B. $(5, \frac{5}{3})$

C. $(3, 0)$

D. $(0, 0)$

$$\rightarrow m = -\frac{1}{3}$$

Solution

Functions

Example 22

If $f(x) = 2x + 5$, find each value.

a. $f(-2) = 2(-2) + 5 = 1$

b. $f(1) + 4 = 2(1) + 5 + 4 = 11$

c. $f(x + 3) = 2(\overbrace{x+3}) + 5 = 2x + 6 + 5 = 2x + 11$

Example 23

If $h(z) = z^2 + 3z - 4$, find each value.

a. $h(-4) = (-4)^2 + 3(-4) - 4 = 0$

b. $h(5a) = (\overbrace{5a})^2 + 3(5a) - 4 = 25a^2 + 15a - 4$

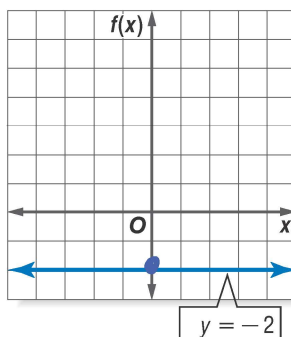
c. $2[h(g)] = 2[g^2 + 3g - 4] = 2g^2 + 6g - 8$

Constant functions

horizontal line // X-axis

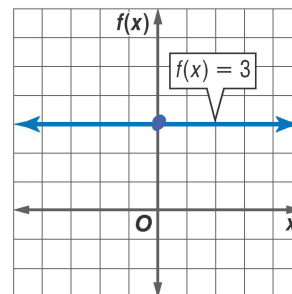
$y = \text{no.}$

$y = -2$



$f(x) = \text{no.}$

$f(x) = 3$



Example 24 EST June 2021

The function g is defined by
 $g(x) = 3x^2 + kx - 8$ and
 $g(-2) = -4$. What is the value of
 $g(-3)$?

Solution

$$g(-2) = -4$$

$$3(-2)^2 + k(-2) - 8 = -4 \quad \text{Shift solve}$$

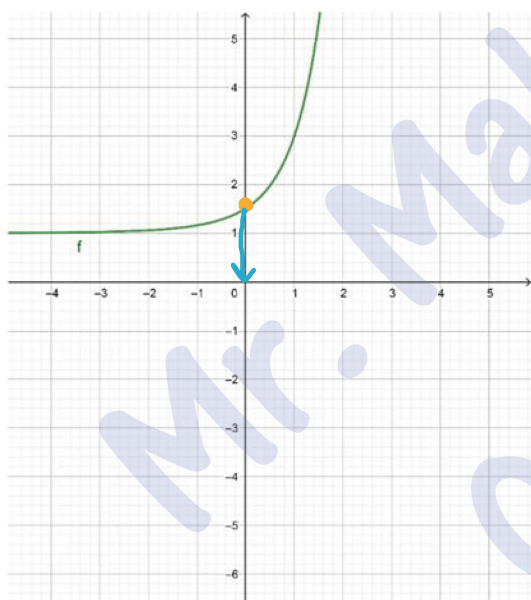
$$k = 4$$

$$g(x) = 3x^2 + 4x - 8$$

$$g(-3) = 3(-3)^2 + 4(-3) - 8$$

Example 25 EST March 2021

$$= 7$$



The graph above represents the curve of an increasing function f .

What is the solution of $f(x) - 1.5 = 0$?

- ☒ A. 0
- ☐ B. 1.5
- ☐ C. 3
- ☐ D. 4.5

$$f(x) = 1.5$$

$$y = 1.5 \rightarrow x ??$$

$$x = 0$$