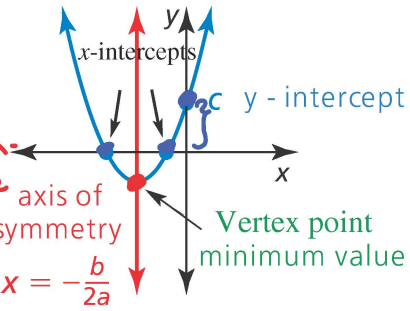
Quadratic functions and its graph

Standard form

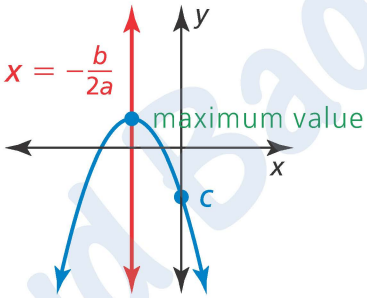
$y = ax^2 + bx + c$ ← y-intercept

a +ve a -ve

$a > 0$



$a < 0$

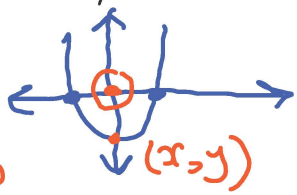


(x, y) → min. or max. value

axis of symm. $-\frac{b}{2a}$

$x = 0$

- The parabola opens up when $a > 0$ and opens down when $a < 0$
- The parabola vertex (x, y) x (abscissa) $= -\frac{b}{2a}$ y (ordinate) → substitute by x in the equation
- The axis of symmetry is $x = -\frac{b}{2a}$ The minimum or maximum value = y
- The midpoint of segment joining th two x-intercepts is x - vertex $= \frac{-b}{2a}$
- y-intercept (initial height) = c
- x-intercepts are called solutions , zeroes , roots
- To find x-intercepts put y = 0 then find the values of x
- The domain is all real numbers
- The range

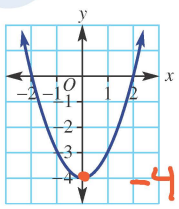


x ??

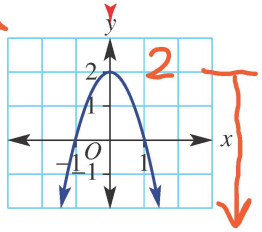
y ??

vertex

∞



$y \geq -4$



$y \leq 2$

-∞

Example 1

Find (a) The axis of symmetry (b) The vertex (c) y-intercept of the graph of $y = x^2 - 4x + 3$

Solution

a. $a = 1$, $b = -4$ $c = 3$

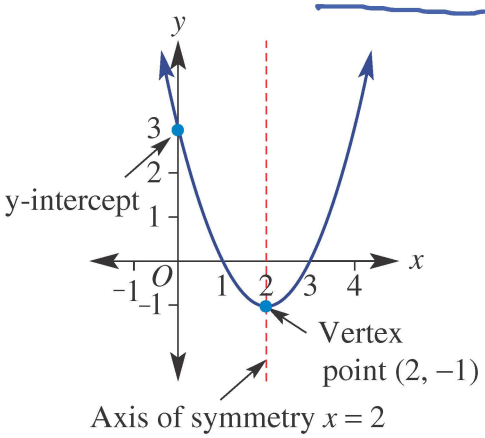
$$x = -\frac{b}{2a} = \frac{4}{2(1)} = 2$$

∴ The axis of symmetry is $x = 2$.

b. $y = (2)^2 - 4(2) + 3 = -1$ minimum value

∴ The vertex is $(2, -1)$

c. ∴ y-intercept = 3



Example 2 EST May 2021

x vertex

What is the abscissa of the vertex of the function $f(x) = 3x^2 - 18x + 4$? (grid-in)

Solution

$a=3 \quad b=-18 \quad c=4$

$x = \frac{-b}{2a} = \frac{18}{2(3)} = 3$

Example 3 EST May 2021

$x \rightarrow y \rightarrow$ range

What is the range of the function $f(x) = x^2 + 4x - 3$?

A. $[-1, +\infty)$
B. $[1, +\infty)$
C. $[7, +\infty)$
D. $[-7, +\infty)$ ✓

Solution

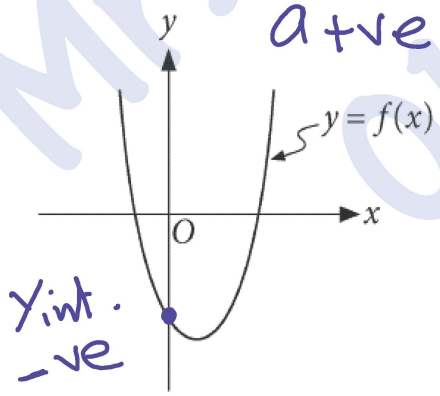
$a=1 \quad b=4 \quad c=-3$

$x = -\frac{b}{2a} = \frac{-4}{2(1)} = -2$

$y = (-2)^2 + 4(-2) - 3 = -7$

Example 4 SAT September 2020 International

a +ve upward



y int. -ve

The graph of the function f is shown. Which of the following could define f?

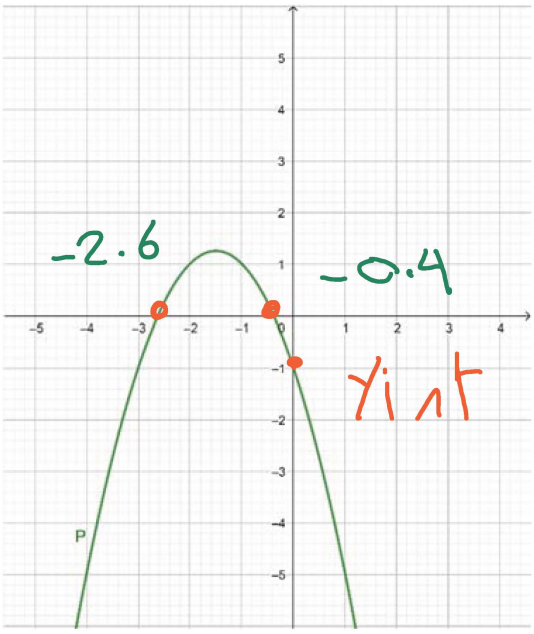
A) $f(x) = -x^2 - 2x - 3$

B) $f(x) = -x^2 + 4x + 3$

C) $f(x) = x^2 - 2x - 3$ ✓

D) $f(x) = x^2 + 4x + 3$

Example 5 EST March 2021



$y_{int} = -1$

$sum = -\frac{b}{a}$

$-2.6 + (-0.4) = \frac{-a}{1}$

$-3 = a$

The curve P above represents function f defined by $f(x) = -x^2 + ax + b$. What are the values of the real numbers a and b?

A. $a = -1 \quad ; b = -3$
B. $a = -3 \quad ; b = -1$
C. $a = -1.5 \quad ; b = -1$
D. $a = -2.5 \quad ; b = -0.5$

X-intercepts form

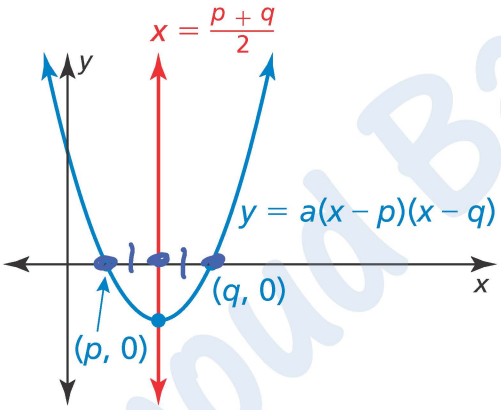
Factored form

change change

$$y = a(x - p)(x - q)$$

2 −5

(x−2) (x+5)



- The parabola opens up when $a > 0$ and opens down when $a < 0$
- The parabola vertex (x , y) $x = \frac{p+q}{2}$ y ⟶ substitute by x in the equation
- The axis of symmetry is $x = \frac{p+q}{2}$ The minimum or maximum value = y
- x-intercepts (solutions , zeroes , roots) = p and q

Example 6

Find (a) The axis of symmetry (b) The vertex (c) x-intercepts of the graph of $f(x) = -2(x + 3)(x - 1)$

$\frac{-3+1}{2} = -1$

Solution

a. $p = -3$ and $q = 1$

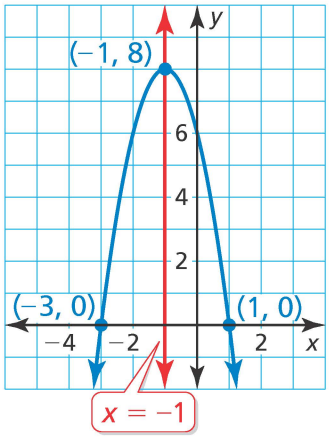
$$x = \frac{p+q}{2} = \frac{-3+1}{2} = -1$$

∴ The axis of symmetry is $x = -1$

b. $y = -2(-1 + 3)(-1 - 1) = 8$ maximum value

∴ The vertex is $(-1, 8)$.

c. ∴ x - intercepts = -3 and 1



Example 7 EST October 2020

The graph (C) of the function $f(x) = 2(x+2)(x-6)$ is a parabola. If the line x=k is the axis of symmetry of the parabola, what is the value of k?

A. 1
B. 2
C. 3
D. 4

Solution

$x_1 = -2 \qquad x_2 = 6$

$x = \frac{-2+6}{2} = 2$

Example 8 EST March 2021

Consider the function f defined by $f(x) = 2(x-3)(x+2)$. What is the ordinate of the vertex of function f ?

A. 2
B. -4.5
C. 2.5
D. -12.5

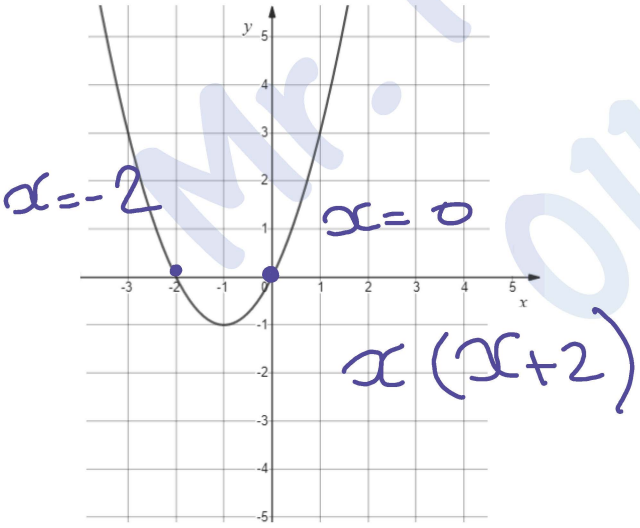
Solution

$x_1 = 3 \qquad x = -2$

$x = \frac{3+(-2)}{2} = \frac{1}{2}$

$y = 2\left(\frac{1}{2}-3\right)\left(\frac{1}{2}+2\right) = -12.5$

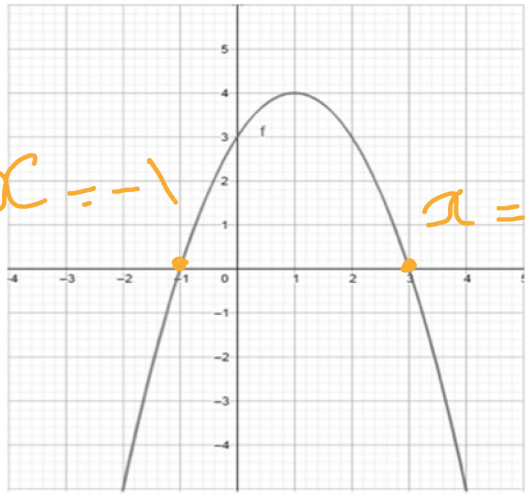
Example 9 EST December 2020



The graph of the function f in the xy-plane above is a parabola. Which of the following expressions defines f while showing the x-intercepts as constants or coefficients?

A. $f(x) = x(x+2)$
B. $f(x) = (x+1)(x+2)$
C. $f(x) = x^2(x+2)$
D. $f(x) = (x+2)^2 - 1$

Example 10 EST October 2020



$x = -1$

$x = 3$

$a \rightarrow -ve$

$-(x+1)(x-3)$

What is the equation of the function **f** graphed above?

A. f(x) = (x+1)(x-3)

B. f(x) = (x-1)(x+3)

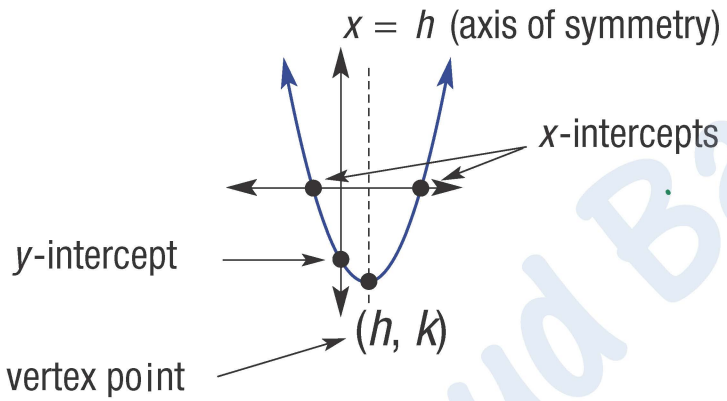
C. f(x) = - (x+1)(x-3)

D. f(x) = - (x-1)(x+3)

vertex form

$$y = a(x - h)^2 + k$$

change → $(x-h)^2$; keep → $+k$



- The parabola opens up when $a > 0$ and opens down when $a < 0$
- The parabola vertex (h, k)
- The axis of symmetry is $x = h$ The minimum or maximum value = $y = k$

$f(x) = x^2 + 1$ ⟶ vertex $(0, 1)$ Opens up minimum value = 1

$f(x) = 3 - x^2$ ⟶ vertex $(0, 3)$ Opens down maximum value = 3

$f(x) = (x + 2)^2$ ⟶ vertex $(-2, 0)$ Opens up minimum value = 0

$f(x) = 4 - (x - 1)^2$ ⟶ vertex $(1, 4)$ Opens down maximum value = 4

$f(x) = 2(x+3)^2 - 5$ ⟶ vertex $(-3, -5)$ Opens up minimum value = -5

Example 11

Find (a) The vertex (b) The axis of symmetry of the graph of $y = (x + 2)^2 - 3$

Solution

$(-2, -3)$

$h = -2$, $k = -3$

$x = -2$

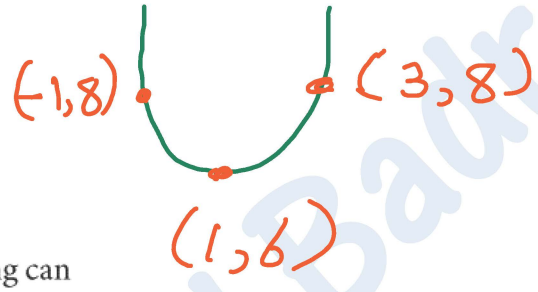
∴ The vertex is $(-2, -3)$.

∴ The axis of symmetry is $x = -2$

Example 12 SAT October 2020 US School Day

The table shows output values, $m(x)$, of quadratic function m for corresponding input values x.

x	$m(x)$
−1	8
1	6
3	8



If a is a positive constant, which of the following can define the function m ?

A) $m(x) = -a(x-1)^2 + 6$

B) $m(x) = -a(x+1)^2 + 6$

C) $m(x) = a(x-1)^2 + 6$

D) $m(x) = a(x+1)^2 + 6$

a +ve

(1,6)

Example 13 EST June 2021

-9

-7

33

9

x	q(x)
-5	-7
-3	-15
-1	-15
1	-7
3	9
5	33

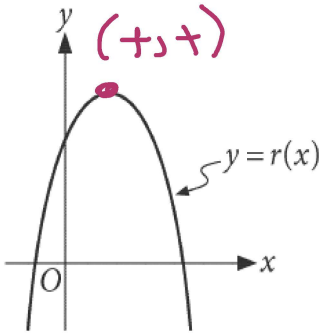
The table above shows several points from the graph of quadratic function q(x). What is q(-9)?

33

y ?? → x = -9

Example 14 SAT October 2020 US

The graph of $y = r(x)$ is shown in the xy-plane.



downward → a −ve

If a, b, and c are positive constants, which of the following could define the function r?

A) $r(x) = -a(x-b)^2 + c$ (b,c)

B) $r(x) = -a(x+b)^2 + c$

C) $r(x) = a(x-b)^2 + c$

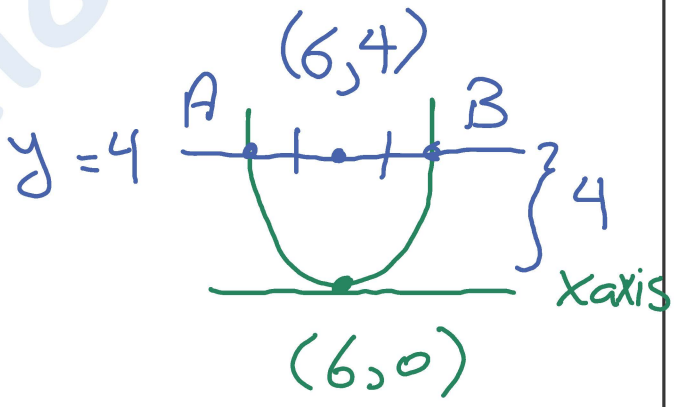
D) $r(x) = a(x+b)^2 + c$

Example 15 EST March 2021

In the xy- plane, the parabola with equation $y = (x-6)^2$ intersects the line with equation $y = 4$ at two points A and B. What is the midpoint of [AB]?

A. (6,0)
B. (2,4)
C. (8,4)
D. (6,4)

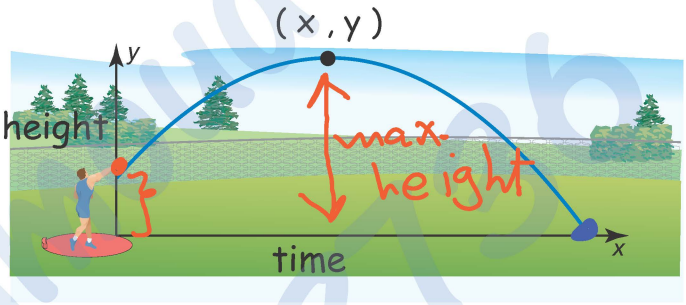
(6,0) vertex



Notes

The parabola vertex (x , y)

1. $x = -\frac{b}{2a}$ or $x = \frac{p+q}{2}$ ⟶ Time reaching to the maximum height or highest point
2. y ⟶ maximum height or highest point
3. y - intercept (x = 0) ⟶ initial height
4. Time of the object hitting the ground ⟶ Put h (t) = 0 then find the values of t



Example 16 EST June 2021

The height of a launched cannonball can be described as a function of time according to the following quadratic equation:

$$h(t) = -2t^2 + 14t + 36$$

What is the maximum height attained by the cannonball?

A. 60.5 ✓
B. 36
C. 9
D. 2

$a = -2 \quad b = 14 \quad c = 36$

$x = \frac{-b}{2a} = \frac{-14}{2(-2)} = \frac{7}{2}$

$y = -2\left(\frac{7}{2}\right)^2 - 14\left(\frac{7}{2}\right) + 36$

$= 60.5$

After how many seconds will the cannonball hit the ground?

A. 2
B. 7
C. 9 ✓
D. 11

$-2t^2 + 14t + 36 = 0$

Mode 5 3

$t = 9$

Example 17 SAT March 2021 International

A ball was thrown into the air from a height of 4 feet (ft). One second after the ball was thrown, it reached a maximum height of 20 ft. The height of the ball can be modeled with a quadratic function, where t is the time, in seconds, since the ball was thrown and $h(t)$ is the height of the ball, in ft. Which function models the height of the ball over time?

A) $h(t) = -16(t-1)^2 + 4$

B) $h(t) = -16(t+1)^2 + 4$

C) $h(t) = -16(t-1)^2 + 20$

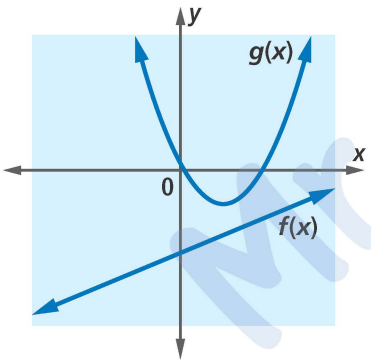
D) $h(t) = -16(t+1)^2 + 20$

(0, 4)

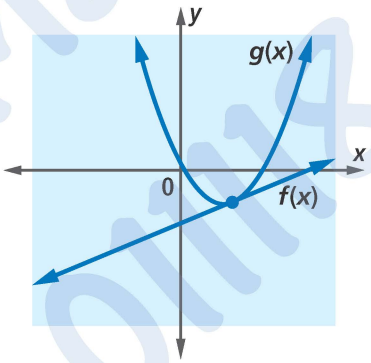
(1, 20)

$h(0) = 4$ $h(1) = 20$

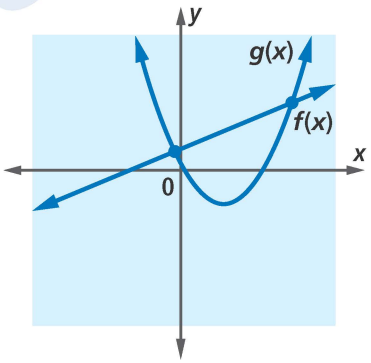
Intersection of a line with parabola



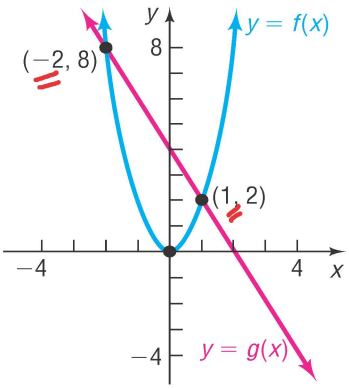
0 intersections
no solution



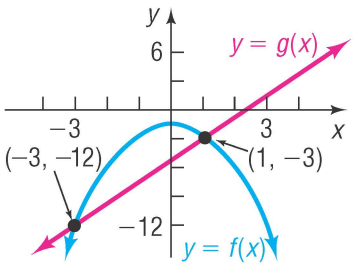
1 intersection
one solution



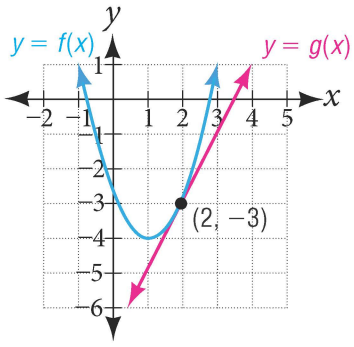
2 intersections
two solutions



The solution set is $\{1, -2\}$



The solution set is $\{1, -3\}$



The solution set is $\{2\}$

Example 18 EST May 2021

Which of the following is a point of intersection between the line with equation $y = 3x + 9$ and the parabola with the equation $y = 2x^2 + 3x + 1$?

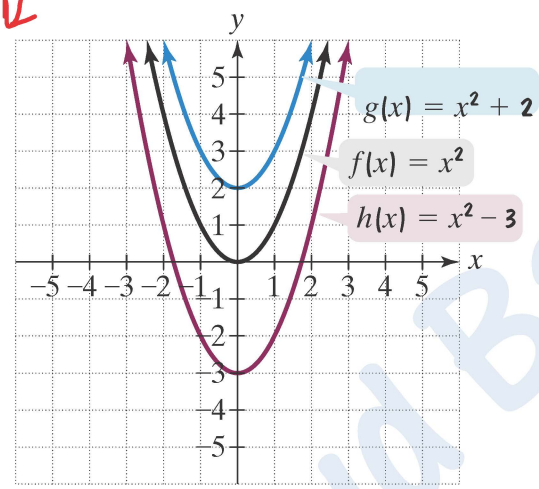
A. $(2, 3)$
B. $(2, 15)$
C. $(-2, 15)$
D. $(-2, 2)$

$y = 3x + 9 \xrightarrow{x=2} y = 15$

$y = 2x^2 + 3x + 1 \xrightarrow{x=2} y = 15$

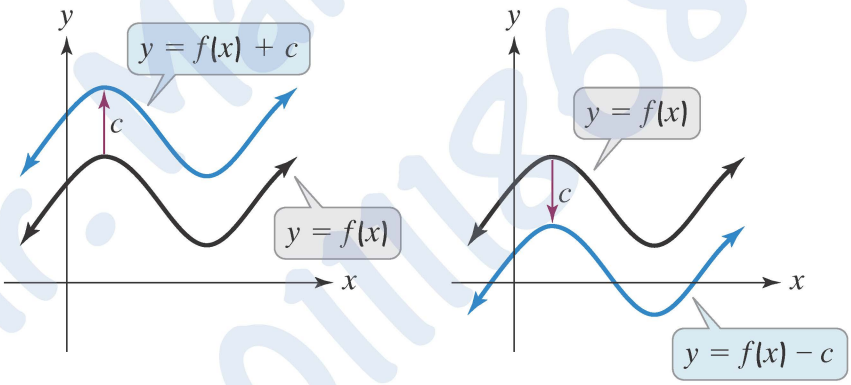
Vertical shifts

$y = a(x-h)^2 + k$



The graph of g $g(x) = x^2 + 2 = f(x) + 2$ shifts the graph of f up 2 units.

The graph of h $h(x) = x^2 - 3 = f(x) - 3$ shifts the graph of f down 3 units.



Horizontal shifts

3 ↓ 3 ↓

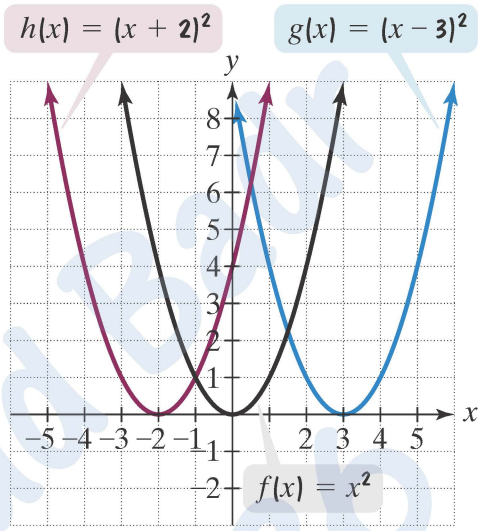
$$g(x) = (x - 3)^2 = f(x - 3)$$

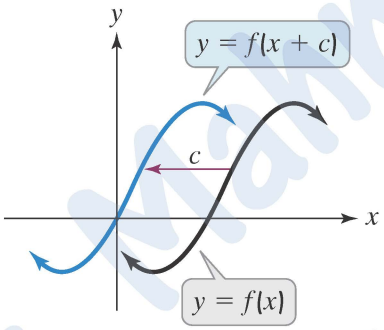
The graph of g shifts the graph of f 3 units to the right.

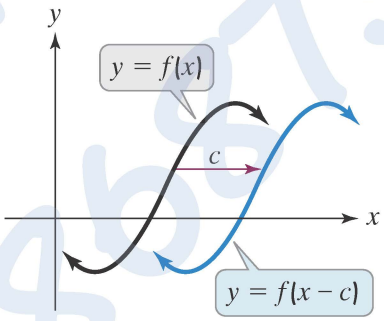
$$h(x) = (x + 2)^2 = f(x + 2)$$

The graph of h shifts the graph of f 2 units to the left.

− 2







Example 19 **EST December 2020**

If $f(x) = (2 - x)(x + 4)$ and $g(x) = f(x - 10)$ are functions whose graphs are parabolas in an xy-plane, what is the y-coordinate of the vertex of the parabola represented by g?

shift to right 10 units

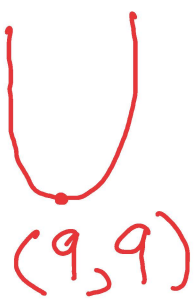
9

$x_1 = 2 \quad x_2 = -4$

$x = \frac{2 + (-4)}{2} = -1$

$y = (2 - (-1))(-1 + 4) = 9$

Vertex $(-1, 9)$



-1 shift to left 1 unit

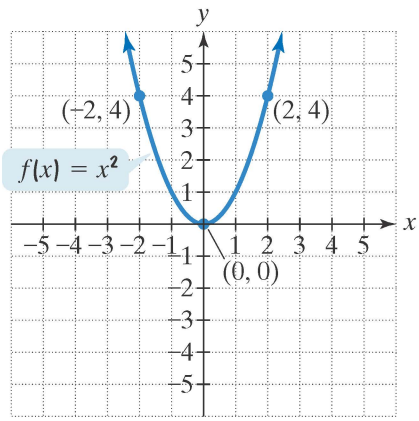
shift down 3 units

Example 20

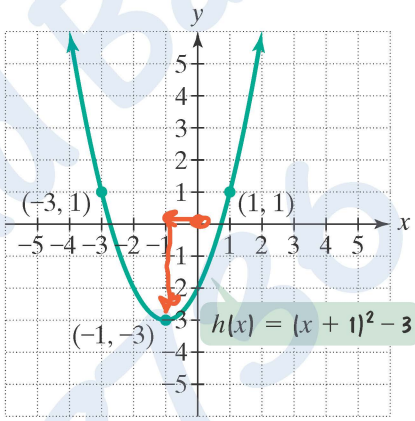
Use the graph of $f(x) = x^2$ to obtain the graph of $g(x) = (x + 1)^2 - 3$.

Solution

The graph of $f(x) = x^2$ with three points identified



The graph of $h(x) = (x + 1)^2 - 3$



Reflection about the x-axis

The graph of $y = -f(x)$ is the graph of $y = f(x)$ reflected about the x-axis.

For each point (x, y) on the graph of $y = f(x)$, the point $(x, -y)$ is on the graph of $y = -f(x)$.