Addition and Subtraction of Polynomials

Example 1

Find the sum.

a. $(5x^2 + 4x + 1) + (2x^2 + 5x + 2)$ $= 7x^2 + 9x + 3$

b. $(4m^2 + 5) + (m^2 - m + 6)$ $= 5m^2 - m + 11$

Solution

Example 2

subtract $2^{nd} - 1^{st}$

Find the difference.

a. $(4x^2 - 3x + 5) - (3x^2 - x - 8)$

b. $(4n^2 + 5) - (-2n^2 + 2n - 4)$

Solution

a) $4x^2 - 3x + 5 - 3x^2 + x + 8 = x^2 - 2x + 13$

b) $6n^2 - 2n + 9$

Example 3 EST December 2020

If $p(x) = x^2 - 7x + 5$ and $q(x) = -3x^3 - 7x^2 + 2x - 5$ which of the following expressions is equal to the difference $p(x) - q(x)$?

A. $4x^3 - 9x + 10$
B. $-3x^3 - 6x^2 - 5x$
C. $-3x^3 - 8x^2 + 9x - 10$
D. $3x^3 + 8x^2 - 9x + 10$

Solution

$P(x) - q(x) = x^2 - 7x + 5 - (-3x^3 - 7x^2 + 2x - 5)$

$= x^2 - 7x + 5 + 3x^3 + 7x^2 - 2x + 5$

$= 3x^3 + 8x^2 - 9x + 10$

Example 4 SAT October 2020 International

$$(x^3 + 2a) + (x^3 + 2b) = cx^3 + 12$$

In the equation above, a, b, and c are constants. If the equation is true for all values of x, what is the value of $a + b + c$?

$a + b + c$

$6 + 2 = 8$

Solution

$2x^3 + 2a + 2b = cx^3 + 12$

$c = 2$ $\quad 2a + 2b = 12 \div (2) \rightarrow a + b = 6$

x^2 ← exponent
← base

Multiplying Polynomials

ConceptSummary Properties of Exponents

For any real numbers x and y, integers a and b:

Property	Definition	Examples
Product of Powers (up x)	$x^a \cdot x^b = x^{a+b}$	$3^2 \cdot 3^4 = 3^{2+4}$ or 3^6; $p^2 \cdot p^9 = p^{2+9}$ or p^{11}
Quotient of Powers ($\frac{}{x}$ down)	$\frac{x^a}{x^b} = x^{a-b}$	$\frac{9^5}{9^2} = 9^{5-2}$ or 9^3; $\frac{b^6}{b^4} = b^{6-4}$ or b^2
Negative Exponent	$x^{-a} = \frac{1}{x^a}$ and $\frac{1}{x^{-a}} = x^a$	$3^{-5} = \frac{1}{3^5}$; $\frac{1}{b^{-7}} = b^7$
Power of a Power	In out $(x^a)^b = x^{ab}$	$(3^3)^2 = 3^{3 \cdot 2}$ or 3^6; $(d^2)^4 = d^{2 \cdot 4}$ or d^8
Power of a Product	$(xy)^a = x^a y^a$	$(2k)^4 = 2^4k^4$ or $16k^4$; $(ab)^3 = a^3b^3$
Power of a Quotient	$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$; $\left(\frac{x}{y}\right)^{-a} = \left(\frac{y}{x}\right)^a$ or $\frac{y^a}{x^a}$	$\left(\frac{x}{y}\right)^2 = \frac{x^2}{y^2}$; $\left(\frac{a}{b}\right)^{-5} = \frac{b^5}{a^5}$
Zero Power	$x^0 = 1$	$7^0 = 1$

$\left(\frac{b}{a}\right)^5$

Example 5 Multiply

a. $(5x^2)(4x^3)$

$= 20x^5$

b. $(-3x^3y^2)(4xy^5)$

$= -12x^4y^7$

c. $\left(\frac{1}{2}a^3b\right)(a^2c^2)(6b^2)$

$= 3a^5b^3c^2$

Example 6 Multiply

a. $5y^3(2y^2 - 7y + 6)$

b. $-4a^3b^7c\left(2ab^2c^4 - \frac{1}{2}a^5b\right)$

Solution

a. $10y^5 - 35y^4 + 30y^3$

b. $-8a^4b^9c^5 + 2a^8b^8c$

Example 7 SAT August 2020 International

The expression $10x(x^2 + 9)$ is equivalent to $ax^b + cx$ where a , b and c are constants. What is the value of abc ?

Solution

$10x^3 + 90x = ax^b + cx$

$a = 10$ $b = 3$ $c = 90$

$a \cdot b \cdot c$

$= 10 \cdot 3 \cdot 90$

$= 2700$

Example 8 Multiply

a. $(x+2)(x-5)$

b. $(3a^2 - b)(a^2 - 2b)$

Solution

a. $x^2 - 3x - 10$

Solution

b. $3a^4 - 6a^2b - a^2b + 2b^2$

$3a^4 - 7a^2b + 2b^2$

Example 9 EST June 2021

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What is the resulting coefficient of x when -2x+3 is multiplied by -3x-2?

A. -9
B. -5
C. 5
D. 6

Solution

$(-2x+3)(-3x-2) = 6x^2 + 4x - 9x - 6$

$6x^2 - 5x - 6$

Example 10 EST December 2020

If $(ax^2 + b)(2x - 1) = cx + 1$ for all values of x, what is the value of $\frac{b}{c}$?

Solution

$2b\cancel{x} = c\cancel{x}$

$2b = 1c$

$\frac{b}{c} = \frac{1}{2}$

Rules

① $(1^{st})^2$ ② $2 \times 1^{st} \times 2^{nd}$ ③ $(2^{nd})^2$

- $(a + b)^2 = a^2 + 2ab + b^2$ — Example: $(x + 4)^2 = x^2 + 8x + 16$
- $(a - b)^2 = a^2 - 2ab + b^2$ — Example: $(2x - 3)^2 = (2x)^2 - 2 \cdot 2x \cdot 3 + 3^2 = 4x^2 - 12x + 9$
- $(a + b)(a - b) = a^2 - b^2$ — Example: $(x + 6)(x - 6) =$

$(x^2 + 2y)(x^2 - 2y) = x^4 - 4y^2$

① $1^{st} \cdot 2^{nd}$ ② $2^{nd} \cdot 2^{nd}$

Example 11 EST March 2020 (SAT)

If $a^2 + b^2 = 20$ and $ab = 8$, then what is $(b - a)^2$?

Solution

$(b - a)^2 = \cancel{b^2} - 2ab + \cancel{a^2}$

$20 - 2(8) = 4$

Example 12 SAT September 2020 US

$2 \cdot 2x \cdot 9$

$= 36x$

$f(x)=(2x+9)^2 = 4x^2+36x+81$

$g(x)=6(6x+10) = 36x+60$

The functions f and g are defined above. Which of the following is equivalent to $f(x)-g(x)$?

Solution

A) $4x^2+21$

B) $4x^2+141$

C) $4x^2-18x+21$

D) $4x^2+72x+141$

$4x^2+36x+81-(36x+60)$

$4x^2+\cancel{36x}+81-\cancel{36x}-60$

$4x^2+21$

Example 13 Multiply

$(5x+3)(2x^2+10x-6) = 10x^3+50x^2-\cancel{30x}+6x^2+\cancel{30x}-18$

$10x^3+56x^2-18$

Dividing polynomials

Rules

▶ $\dfrac{a+b}{c} = \dfrac{a}{c} + \dfrac{b}{c}$

▶ $\dfrac{a-b}{c} = \dfrac{a}{c} - \dfrac{b}{c}$

Example 14 Divide

a. $\dfrac{3x^4-6x^3+9x}{3x}$

b. $(10c^3d - 15c^2d^2 + 2cd^3) \div (5c^2d^2)$

Solution

a. x^3-2x^2+3

Solution

b. $\dfrac{10c^3d^1-15c^2d^2+2c^1d^3}{5c^2d^2}$

$= 2cd^{-1}-3+\frac{2}{5}c^{-1}d$

$= \dfrac{2c}{d} - 3 + \dfrac{2d}{5c}$