MATH TEST KEY POINTS

TEST 5

The **Key Points** section reviews many of the concepts you will encounter on the SAT Math Practice Tests. Each Key Point corresponds to one or more test questions. Key Points with a **nc** next to them correspond to "no calculator" questions, while Key Points with a **c** next to them correspond to "calculator" questions. The numbers next to the **nc** and **c** labels tell you which question each Key Point relates to.

1. Equation of a Line (c1, c4)

There are several ways to write the equation of a linear function.

- **Point-Slope Form:** $y - y_1 = m(x - x_1)$.
  A line in this form has slope m and includes the point (x_1, y_1).
- **Slope-Intercept Form:** $y = mx + b$. A line in this form has slope m and includes y-intercept b (the point $(0, b)$).
- **Standard Form:** $Ax + By = c$.
- **Intercept Form:** $\frac{x}{a} + \frac{y}{b} = 1$. A line in this form has x-intercept a and y-intercept b.

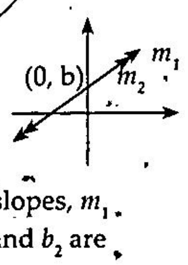
2. Parallel and Perpendicular Lines(c28)

- Two lines are parallel if their slopes, m, are identical.
- Two lines are perpendicular if their slopes, m_1 and m_2 are negative reciprocals.

TIP :

$m_1 \cdot m_2 = -1$

so, $m_1 = -\frac{1}{m_2}$



- Two lines are identical if their slopes, m_1 and m_2 and the y-intercepts, b_1 and b_2 are identical.

3. Factoring Quadratics (c17)

A quadratic trinomial of the form $ax^2 + bx + c$ for which $a \neq 1$ can be factored in the following way:

If $ax^2 + bx + c = pqx^2 + (ps + qr)x + rs$, where $a = pq$, $b = ps + qr$ and $c = rs$, then, the factored form is $(px + r)(qx + s)$.

Quadratics can be solved by factoring or using the quadratic equation:

Remember:

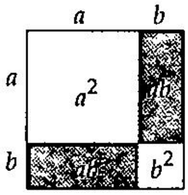
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4. Definitions of a Function (c26)

- A **function** is a set of ordered pairs with a consistent relation between the two values of each pair. Often times, a function is denoted as $y = f(x)$.
- The **domain** of a function is the set of all the first coordinates of the ordered pairs.
  The domain of $y = f(x)$ is the set of x-coordinates on the graph of the function.
- The **range** of a function is the set of all the second coordinates of the ordered pairs.
  The range of $y = f(x)$ is the set of y-coordinates on the graph of the function.

5. Factorization of Squares (nc19)

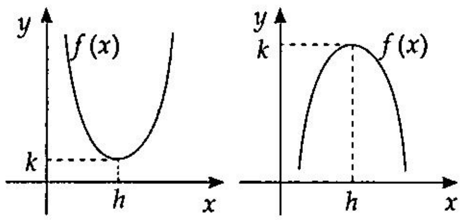
$a^2 - b^2 = (a + b)(a - b)$
$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$



6. Interpreting a Quadratic Function (c35)

The graph of a quadratic function $y = a(x - h)^2 + k$ has its minimum value k at the vertex (h, k) if $a > 0$ (the parabola opens upward).

$y = a(x - h)^2 + k$ has its maximum value k at the vertex (h, k) if $a < 0$ (the parabola opens downward).



7. Operations with Exponents (c11)

The operations of exponents are summarized below. Assume all bases and denominators are nonzero.

$(a^m)(a^n) = a^{m+n}$	$(a^m)^n = a^{mn}$
$(ab)^n = a^n b^n$	$\left(\dfrac{a}{b}\right)^0 = \dfrac{a^0}{b^0} = 1$
$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$	$\left(\dfrac{a}{b}\right)^{-n} = \dfrac{a^{-n}}{b^{-n}} = \dfrac{b^n}{a^n}$
$\dfrac{a^m}{a^n} = a^{m-n}$	$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

8. Rationalizing the Denominator (c9)

$a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$ are called conjugates☆ of one another. Their product is a difference of squares. If we have a fraction with a radical in the denominator, we are required to manipulate the expression so that the radical in the denominator is removed.

Remember☼:

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{(\sqrt{a}+\sqrt{b})(\sqrt{a}+\sqrt{b})}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})}$$
$$= \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a})^2-(\sqrt{b})^2} = \frac{a+2\sqrt{ab}+b}{a-b}$$

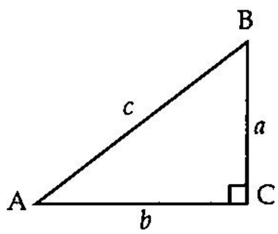
9. Simplifying Complex Numbers (c7)

When a complex number is in the denominator of a fraction, it must be rationalized in the same way as a radical in the denominator. This is accomplished by multiplying the complex number by its conjugate. The **conjugate** of the complex number $a + bi$ is $a - bi$. When conjugate are multiplied, they give as product the difference of two squares:

Remember☼:

$$(a + bi)(a - bi) = a^2 - b^2(i)^2 = a^2 + b^2$$

10. Pythagorean Theorem (nc15, nc17)

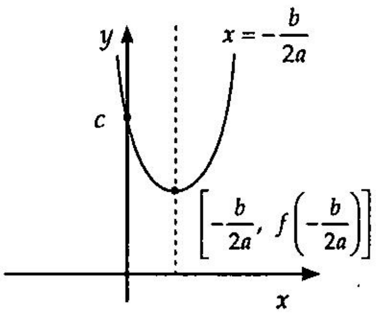


If a, b, c are the sides of the right triangle and a and b are adjacent to the right angle, then the following equation holds:

$$c^2 = a^2 + b^2$$

11. Graphing Quadratic Function (nc14)

The graph of a quadratic function is a parabola. It is either the standard form or the vertex form of the equation:



- **Standard Form:** $y = ax^2 + bx + c$

  The vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

  The axis of symmetry is $x = -\frac{b}{2a}$.

  The y-intercept is $(0, c)$

  The x-intercepts are $\left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, 0\right)$.

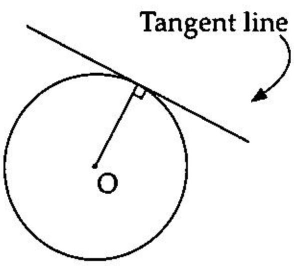
- **Vertex Form:** $y = a(x-h)^2 + k$

  The vertex is (h, k).

  If x-intercepts are $(p, 0)$ and $(q, 0)$, then $h = \frac{p+q}{2}$.

12. A Tangent to a Circle (c34)

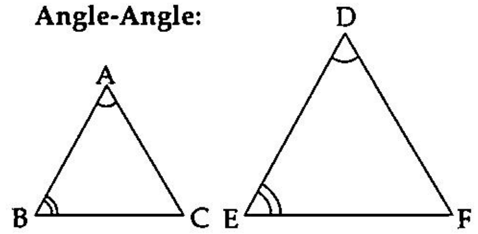
A tangent to a circle is a straight line which touches the circle at only one point. A tangent to a circle forms a right angle with the circle's radius at the point of contact with the tangent.



13. Similarity (nc13)

- <u>AA</u>: Two pairs of angles are the <u>same size</u>.*

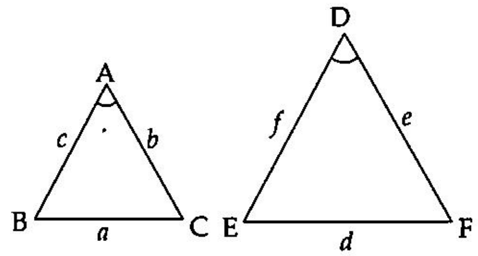
Angle-Angle:



If $\angle A = \angle D$ and $\angle B = \angle E$, then $\Delta ABC \sim \Delta DEF$

- <u>SAS</u>: One pair of angles are equal and the sides that are adjacent to those angles on the two triangles are proportional.

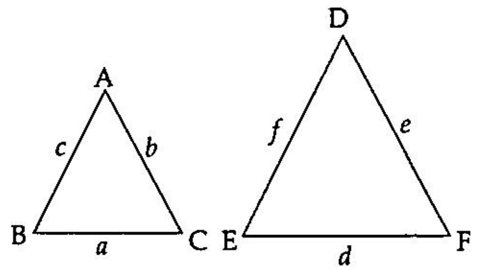
Side-Angle-Side:



If $\angle A = \angle D$ and $\frac{c}{f} = \frac{b}{e}$, then $\Delta ABC \sim \Delta DEF$

- <u>SSS</u>: Three pairs of corresponding sides on the triangles are proportional.

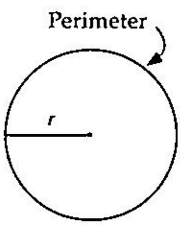
Side-Side-Side:



If $\frac{a}{d} = \frac{c}{f} = \frac{b}{e}$, then $\Delta ABC \sim \Delta DEF$.

14. Circles (c34)

Let r be the radius of the circle.
$\pi \approx 3.14$
Diameter = $2r$
Area of circle $A = \pi r^2$
Circumference of circle $C = 2\pi r$



Remember:

Equation of circle*: $(x - h)^2 + (y - k)^2 = r^2$

** where (h, k) is the coordinate of the center and r is the radius.*

15. Radian Conversions (c23)

A radian measure of π is equivalent to the radian measure of a semi-circle. As the degree measure of a circle is 360°, the following conversion factors can be used to convert radians into degrees and vice versa:

Remember:

$$1 \text{ radian} = \frac{180}{\pi}\text{degrees, } 1 \text{ degree} = \frac{\pi}{180}\text{radians}$$

16. Special Products (nc18)

If $xy = a$, $yz = b$, and $xz = c$, then $(xyz)^2 = abc$,
$xyz = \pm\sqrt{abc}$

17. Product of Complex Numbers (c7)

Any number of the form $a + bi$, where a and b are real numbers, $b \neq 0$ and i is the imaginary unit, is called a **complex number**. Given two complex numbers $a + bi$ and $c + di$, their product is as follows:

Remember:

$$\begin{aligned}(a + bi)(c + di) &= ac + (bc + ad)i + bdi^2 \\ &= ac - bd + (bc + ad)i\end{aligned}$$

18. Arcs, Sectors, and Central Angles

If r is the radius of sector A (shown below), s is the arc length of the sector, and θ is the measurement of the central angle in radians, then the area of a sector A is given as follows:

$$A = \frac{1}{2}rs$$

The relationship between a sector's radius (r) and its arc length (s) is shown below:

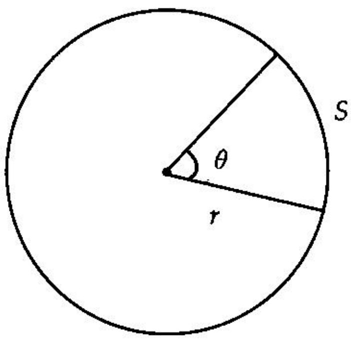
$$s = r(\theta)$$

If r and θ are given, and if θ is in degrees, then the area of a sector A is given as follows:

$$A = \frac{1}{2}r^2\theta$$

$$A = \frac{1}{2}r^2\theta \cdot \left(\frac{\pi}{180}\right)$$

$$A = \frac{\theta}{360}\pi r^2$$



19. Exponential Function (c14)

An exponential function is a function of the form $f(x) = b^x$ in which the input variables x occurs as an exponent. A function of the form $f(x) = a \cdot b^x$ is also considered an exponential function.

To solve, simply take the **log** or the **natural log** of the exponential function to bring the exponents down:

$$\ln e^x = x \cdot \ln e = x$$

$$\log 2^x = x \cdot \log 2$$

20. Rates (c31)

Rates are a measure of the change in some quantity per unit time. Common rates include speed and work. Average speed is calculated using the formula:

Remember:

$$Average\ Speed = \frac{total\ distance}{total\ time}$$